# Mathematical Paradoxes in Finance: St. Petersburg and Simpson's

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## 1 Introduction

Mathematical paradoxes often reveal the counterintuitive nature of abstract concepts, especially when applied to real-world fields like finance. Here, we explore two such paradoxes: the St. Petersburg Paradox, which challenges expected value in decision-making, and Simpson's Paradox, which exposes flaws in aggregated data analysis. Both have profound implications for investors, traders, and quants in areas like risk assessment, trading strategies, and behavioral finance.

## 2 The St. Petersburg Paradox

The St. Petersburg Paradox, named after the city where it was discussed in the 18th century, is a probability puzzle involving a hypothetical lottery game. It demonstrates how infinite expected value can clash with rational human behavior.

### 2.1 The Game Setup

Consider a fair coin tossed repeatedly until the first heads appears. Let $n$ be the number of tosses until the first heads (so $n \geq 1$). The payout $W(n)$ is $2^n$ dollars. The probability $P(n)$ of the first heads on the $n$-th toss is $\frac{1}{2^n}$.

The expected value $EV$ of the game is:

$$EV = \sum_{n=1}^{\infty} P(n) \cdot W(n) = \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot 2^n = \sum_{n=1}^{\infty} 1 = \infty.$$

This infinite sum arises because each term simplifies to 1, and there are infinitely many terms.

### 2.2 The Paradox

Despite the infinite expected value, no rational person would pay a large entry fee (e.g., \$100) to play, as the game could yield very low payouts with high probability (e.g., 50% chance of just \$2). This discrepancy highlights limitations in expected monetary value as a decision criterion.

### 2.3 Link to Finance

In finance, this paradox underpins utility theory and risk aversion. Investors often prioritize utility (diminishing marginal satisfaction from wealth) over raw expected value. For example: - Option pricing and insurance premiums must account for rare, high-impact events (like black swan payouts). - In crypto trading, high-volatility assets (e.g., meme coins) mimic this: theoretically infinite upside, but practical aversion due to ruinous downside.

Daniel Bernoulli resolved it via logarithmic utility: $U(w) = \log(w)$, yielding a finite expected utility of about \$2–\$4.

# 3 Simpson's Paradox

Simpson's Paradox (formalized in 1951) occurs when trends in subgroups of data reverse upon aggregation, due to lurking variables or unequal sample sizes. It's a cautionary tale for statistical inference.

## 3.1 A Finance Example

Suppose we compare two investment funds, A and B, over two market regimes: Bull (high returns) and Bear (low returns). Success rate is the proportion of profitable trades.

| Fund | Bull Market (Success) | Bear Market (Success) | Combined Success |
|---|---|---|---|
| A | $90/100 = 90\%$ | $10/100 = 10\%$ | $100/200 = 50\%$ |
| B | $1/10 = 10\%$ | $80/90 \approx 89\%$ | $81/100 = 81\%$ |

Table 1: Simpson's Paradox in Fund Performance

In subsets: Fund A excels in bull (90%), B in bear (89%). Aggregated: B outperforms (81% vs. 50%).

## 3.2 The Math

Success rate:

$$r = \frac{s}{n}$$

, where $s$ is successes, $n$ is trials. Combined for A:

$$r_A = \frac{90 + 10}{100 + 100} = \frac{100}{200} = 0.5.$$

For B (unequal weights):

$$r_B = \frac{1 + 80}{10 + 90} = \frac{81}{100} = 0.81.$$

The reversal stems from B's larger bear-market sample, weighting the "strong" performance more.

## 3.3 Link to Finance

This paradox plagues financial data: - Backtesting algos: A strategy shines in volatile crypto but fails overall due to regime shifts. - Portfolio reporting: Sub-asset classes (e.g., stocks vs. bonds) show trends that invert aggregated (2008 crisis example). - Causal inference tools like stratified sampling or regression are essential to avoid it in DeFi yield optimization or credit risk models.

# 4 Conclusion: Implications for Finance and Crypto

These paradoxes remind us that math, while powerful, demands context. The St. Petersburg Paradox urges behavioral adjustments in valuation models, while Simpson's warns against blind aggregation in big data era trading. In quantitative finance and crypto courses, they teach robust analysis—preventing overconfidence in models that ignore human psychology or data pitfalls.

For further reading, explore utility functions in Bernoulli's work or causal diagrams for Simpson's. These concepts can transform how we approach risk in volatile markets.