Assumptions in Quantitative Finance

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Intoduction

Quantitative models rely on mathematical structure to make market phenomena tractable. Every model, from Black–Scholes to modern market microstructure frameworks, is powered by a set of assumptions. These assumptions are often implicit, rarely challenged, and frequently responsible for large failures when they break.

This research note outlines the major classes of assumptions used in quantitative finance, why they are convenient, how they fail, and how they behave differently in decentralized markets.

1. Distributional Assumptions

Many models assume that returns follow Gaussian or elliptic distributions. This assumption enables closed-form pricing, risk metrics such as VaR, and quadratic utility maximization. However, empirical evidence shows that returns exhibit fat tails, skewness, volatility clustering, and jumps.

- Black–Scholes assumes log-normal returns and constant volatility.
- Portfolio theory assumes multivariate normality and stable covariance.
- Risk models assume finite variance and tail independence.

Empirically, tail risk violates these assumptions, leading to underestimation of extreme drawdowns.

2. Dynamical Assumptions

A large class of quantitative models assume that asset prices evolve as continuous-time Itô diffusions with smooth sample paths. The textbook formulation is the geometric Brownian motion (GBM):

$$\mathrm{d}S_t = \mu S_t \,\mathrm{d}t + \sigma S_t \,\mathrm{d}W_t,$$

where W_t is a Wiener process, μ is constant drift, and σ is constant volatility.

This assumption delivers powerful tractability: log-normal returns, closed-form option prices, absence of arbitrage through Girsanov transformations, and analytically manageable hedging rules. However, a substantial body of empirical work shows that GBM dynamics are violated in several dimensions.

Absence of jumps and discontinuities. GBM sample paths are continuous, yet real-world prices exhibit jumps due to macro news, liquidation cascades, and endogenous market stress. Empirical studies such as Merton [24] and Kou [25] show that returns contain discontinuities that cannot be captured by diffusion-only processes. Jump-diffusion models extend GBM to

$$\mathrm{d}S_t = \mu S_t \,\mathrm{d}t + \sigma S_t \,\mathrm{d}W_t + S_{t^-} \,\mathrm{d}J_t,$$

where J_t is a compound Poisson jump process. These models improve fit to fat tails and skewness but introduce additional parameters that are difficult to identify and calibrate.

Volatility is not constant. One of the most widely documented empirical facts is that volatility is stochastic and mean-reverting. The Heston model [26] replaces constant σ with a diffusion:

$$\mathrm{d}v_t = \kappa(\theta - v_t)\,\mathrm{d}t + \xi\sqrt{v_t}\,\mathrm{d}Z_t,$$

with v_t as instantaneous variance. This captures volatility clustering and skew, but introduces calibration instabilities: multiple parameter sets often fit market data equally well.

More recent work on rough volatility [27] shows that volatility behaves like a fractional process with Hurst exponent $H < 0.5$:

$$v_t = v_0 + \int_0^t (t-s)^{H-0.5}\mathrm{d}W_s,$$

which captures long memory, microstructure noise, and persistent irregularity. Rough models fit implied volatility surfaces exceptionally well but are computationally expensive and sensitive to estimation error.

Violation of scaling laws and fat tails. Empirical distributions of returns show heavy tails and excess kurtosis [28, 29]. Continuous diffusions imply thin tails and exponential moments that do not match historical data. Stable distributions, generalized hyperbolic distributions, and variance-gamma processes attempt to address these stylized facts, but they again require calibration to ill-posed inverse problems.

State-dependent and regime-dependent dynamics. In real markets, volatility, drift, and even liquidity regimes change depending on market state. Regime-switching models [30] incorporate transitions governed by a Markov chain:

$$\mathrm{d}S_t = \mu_{X_t} S_t\,\mathrm{d}t + \sigma_{X_t} S_t\,\mathrm{d}W_t,$$

where X_t is a latent regime variable. These models better capture volatility clustering and crises, yet suffer from parameter instability and unobservable state problems.

Non-equilibrium dynamics and feedback effects. Diffusion models assume exogenous shocks. In reality, order flow, liquidity demand, and funding constraints generate endogenous volatility. Empirical microstructure work [31] shows that price changes are driven by long-memory order flow, metaorders, and self-exciting dynamics rather than Gaussian noise. This violates independence and martingale assumptions underlying classical diffusions.

In summary, diffusion-based dynamical assumptions simplify mathematics but fail to describe key empirical properties of financial markets: jumps, fat tails, long memory, feedback loops, and regime shifts. Modern extensions provide better realism but introduce calibration fragility, parameter instability, and computational complexity.

3. Market Microstructure Assumptions

Market making and execution models (such as Avellaneda–Stoikov) rely on:

- Poisson order arrivals,
- stationary intensity,
- symmetric order flow,
- continuous liquidity,
- frictionless rebalancing.

In practice, order flow is bursty, autocorrelated, and adversarial. Liquidity evaporates during stress, and execution impacts future flow through feedback loops.

4. Correlation and Dependence Assumptions

Classical credit and portfolio models assume stable correlation structures. Merton-type credit models treat defaults as conditionally independent. Gaussian copulas assume dependence can be summarized through linear correlation.

However:

- Correlations spike during crises.
- Dependence becomes nonlinear.
- Tail dependence increases sharply.

These shifts create model risk that is invisible under normal conditions.

5. Funding and Market Access Assumptions

Many models assume:

- frictionless borrowing and lending,
- continuous rebalancing,
- unlimited liquidity at the marginal price,
- no constraints on leverage or shorting.

In reality, none of these hold. Transaction costs, collateral requirements, and slippage generate nonlinearities that accumulate over time.

6. Crypto-Specific Assumption Failures

Decentralized markets amplify classical model fragilities and introduce new failure modes absent in traditional finance. Because execution, settlement, and price formation occur on a shared blockchain rather than in a centralized matching engine, crypto markets exhibit structural discontinuities that break many continuous-time and equilibrium assumptions.

Discreteness of block times and settlement latency. Traditional models assume continuous price updating and immediate trade settlement. On-chain execution depends on block production, which introduces discrete time increments with stochastic latency. Work by Lehar and Parlour (2021) shows that block-time granularity creates time-varying liquidity gaps, stale pricing, and asynchronous execution across protocols.[18] Thus, assumptions of continuous diffusion or continuous hedging are systematically violated.

Oracle update frequency and data latency. Most DeFi protocols rely on oracles such as Chainlink or TWAP-based pricing. Oracles update at discrete intervals, producing latency and smoothing that disconnects protocol states from true market prices. Zhang et al. (2021) demonstrate how oracle delay creates exploitable windows for manipulation and destabilizes lending protocols.[19] This violates the assumption of instantaneous or frictionless information flow.

Gas costs, congestion, and endogenous execution risk. Execution price is not only a function of market liquidity but also gas costs, which vary with network congestion. Amini, Kane, and Janson (2022) show that gas auctions introduce endogenous priority markets and nonlinear execution risk.[20] This contradicts models with frictionless trading and zero-cost rebalancing.

Liquidation engine efficiency and discrete collateral updates. DeFi lending protocols rely on liquidation bots rather than centralized risk desks. Liquidations occur only when a liquidator submits a transaction, which depends on: block times, gas fees, oracle updates, and arbitrage incentives. Gudgeon et al. (2020) document how delayed liquidations generate cascade failures during volatility spikes.[21] This violates assumptions of continuous margining and immediate collateral enforcement.

Cross-chain inconsistency and fragmented state. Many trading strategies rely on assets bridged across multiple chains. Finality, latency, and liquidity conditions differ sharply across ecosystems. Garcia and Jha (2022) show that cross-chain bridges behave like settlement systems with inconsistent states and probabilistic guarantees.[22] Any model assuming a unified price process or consistent settlement layer fails under fragmentation.

Validator-level execution risk and consensus fragility. Execution depends on block producers who may censor, reorder, or delay transactions. MEV (Maximal Extractable Value) creates predictable priority flows that move prices, invalidate hedges, and cause discontinuities in PnL paths. Qin et al. (2021) prove that MEV introduces structural arbitrage path-dependence and breaks assumptions of martingale pricing and fair execution.[23] This contradicts models that assume competitive, non-adversarial execution environments.

MEV routing and non-equilibrium price formation. Because transactions compete for block inclusion, price updates are path dependent. Jumps in state occur when large liquidations, arbitrages, or liquid staking rebalances are prioritized within a block. This means that even the basic assumption "prices evolve continuously" fails: crypto prices evolve in *bundles of discrete state transitions* rather than smooth diffusions.

Crypto is not a continuous-time market but a hybrid computational-financial system. Models that ignore block-level discreteness, oracle latency, MEV flows, and cross-chain fragmentation systematically underestimate risk and misinterpret observed price dynamics.

7. Why Assumptions Fail

Model failures rarely come from algebraic mistakes. They come from using a model outside the domain where its assumptions hold. The main failure modes documented in the literature can be grouped into five families.

Overfitting and regime dependence. Overfitting occurs when a model is tuned too closely to a particular sample, capturing noise rather than structure. Such models perform well in sample but degrade sharply out of sample or in live trading.[7] In quantitative finance this is exacerbated by:

- short histories for many assets,
- structural changes in microstructure and regulation,
- the ability to search over extremely large model spaces.

Industry studies on model risk management stress that many so called model failures are instances of *misuse*: applying a model in a regime different from the one it was calibrated on, or under data conditions for which it was never designed.[8]

Structural breaks and nonstationarity. Most pricing and risk models rely on some form of stationarity: that return distributions, volatilities, or correlations are stable enough for historical estimates to remain informative. Empirical work on financial time series shows persistent violations of this assumption through volatility clustering, regime shifts, and structural breaks.[29] Regime-switching and mixture models can partially capture this,[10] but in practice many production models still assume a single stable regime, leading to underestimation of tail events when the environment changes.

Endogenous risk and amplification mechanisms. Classical risk models often treat shocks as exogenous. However, a growing literature emphasizes *endogenous risk*: shocks that are generated and amplified within the financial system through the interaction of leveraged institutions, risk management constraints, and common trading strategies.[11, 12] Brunnermeier and Pedersen show how funding constraints and margin requirements can create liquidity spirals, where a deterioration in market liquidity tightens funding, which then forces asset sales that further reduce liquidity.[13] In such settings, assumptions of independent shocks or constant liquidity are systematically violated exactly when they matter most.

Ignoring feedback loops and reflexivity. Many equilibrium models treat prices as passive reflections of fundamentals. Soros and later authors describe *reflexivity*: expectations influence prices, prices influence balance sheets and funding conditions, and those in turn feed back into expectations.[14, 15] Feedback loops can be stabilizing or destabilizing; the latter generate bubbles, crashes, and persistent deviations from fundamental value. Models that assume that agents are small, price-taking, and do not affect the system state underestimate these reflexive dynamics and mismeasure risk during boom and bust cycles.

Equilibrium tools in non-equilibrium environments. Standard asset-pricing frameworks rely on some notion of equilibrium: a fixed point where supply equals demand and no agent wishes to deviate. Farmer and Geanakoplos argue that while equilibrium theory is powerful, applying equilibrium models to markets that are constantly out of equilibrium, with balance sheet constraints, defaults, and institutional frictions, can be misleading.[16] Similarly, work on general equilibrium with financial fragility shows that small shocks can push the system far from equilibrium because of default and non-linear constraints.[17] In such environments, using stationary equilibrium models for risk management hides the very dynamics that generate crises.

Assumptions are not truths. They are modelling choices that trade realism for tractability. Understanding where and why they fail, overfitting, structural breaks, endogenous amplification, feedback loops, and non-equilibrium dynamics, is essential for assessing model risk and systemic fragility.

8. Conclusion

Quant models do not fail because mathematics is wrong. They fail when the world violates the assumptions that made the math convenient. Robust modelling requires identifying where assumptions live, when they hold, and how they collapse under stress.

References

[1] Taleb, N. (2007). *The Black Swan*. Analysis of fat tails and distributional fragility.

[2] Avellaneda, M. and Stoikov, S. (2008). *High Frequency Trading in a Limit Order Book*. Optimal quoting under Poisson order flow assumptions.

[3] Hull, J. (2018). *Options, Futures, and Other Derivatives*. Classical derivations under diffusion assumptions.

[4] Cont, R. (2001). Empirical properties of asset returns. Demonstrates fat tails and volatility clustering.

[5] Merton, R. (1974). *On the Pricing of Corporate Debt*. Structural credit modeling under independence assumptions.

[6] Gatheral, J., et al. (2018). *Rough Volatility*. Breakdown of classical smooth volatility assumptions.

[7] Harvey, C. R., Liu, Y., and Zhu, H. (2016). *... and the Cross Section of Expected Returns*. Review of Financial Studies, 29(1), 5–68. Demonstrates how overfitting and data mining distort empirical finance results.

[8] PricewaterhouseCoopers (2013). *Model Risk Management: The New Imperative*. PwC Financial Services Report. Highlights industry-wide failures caused by misuse of models outside their valid regimes.

[9] Cont, R. (2001). *Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues*. Quantitative Finance, 1(2), 223–236. Foundational reference for nonstationarity, fat tails, and volatility clustering.

[10] Gorham, M. and Wahal, S. (2021). *Regime Switching Models in Financial Markets*. Two Sigma Research Commentary. Discusses how regime changes break static volatility and correlation assumptions.

[11] Danielsson, J., Shin, H. S., and Zigrand, J. P. (2002). *The Impact of Risk Regulation on Price Dynamics*. Journal of Banking and Finance, 26(7), 1253–1273. Introduces endogenous risk: shocks amplified by feedback loops within the system.

[12] Danielsson, J. and Shin, H. S. (2012). *Endogenous Risk*. In: The Oxford Handbook of Quantitative Asset Management. Provides the full theoretical framework for endogenously generated financial instability.

[13] Brunnermeier, M. K. and Pedersen, L. H. (2009). *Market Liquidity and Funding Liquidity*. Review of Financial Studies, 22(6), 2201–2238. Shows how liquidity spirals form when funding constraints interact with market liquidity.

[14] Soros, G. (2009). *The Crash of 2008 and What It Means*. PublicAffairs. Presents the reflexivity concept in financial markets.

[15] Ehrets, D. and Franke, J. (2014). *Reflexivity in Finance: A Critical Review*. Journal of Economic Issues, 48(1), 29–52. A modern academic treatment of reflexive feedback loops.

[16] Farmer, J. D. and Geanakoplos, J. (2009). *The Virtues and Vices of Equilibrium*. Complexity, 14(3), 11–38. A critique of applying equilibrium models to unstable, path-dependent systems.

[17] Tsomocos, D. (2003). *Equilibrium Analysis, Banking, and Financial Fragility*. Bank of England Working Paper 183. Shows how small shocks lead to large systemic effects in non-linear equilibrium structures.

[18] Lehar, A., Parlour, C., and Zoican, M. (2021). *On the Latency of Decentralized Exchanges*. Working Paper.

[19] Zhang, R., Xiong, W., and Wang, D. (2021). *DeFi Security: Oracle Attacks, Manipulation, and Mitigation*. arXiv:2106.10740.

[20] Amini, H., Kane, E., and Janson, A. (2022). *Priority Gas Auctions and the Economics of Blockchain Congestion*. Working Paper.

[21] Gudgeon, L., Perez, D., Harz, D., Livshits, B., and Danezis, G. (2020). *The Decentralized Finance (DeFi) Attack Surface*. Proceedings of IEEE Security and Privacy Workshops.

[22] Garcia, D. and Jha, A. (2022). *Bridges, Finality, and Cross-Chain Fragility.* Working Paper.

[23] Qin, K., Zhou, L., and Gervais, A. (2021). *Quantifying Blockchain Extractable Value: How Dark Is the Forest.* IEEE Symposium on Security and Privacy.

[24] Merton, R. (1976). *Option Pricing When Underlying Stock Returns Are Discontinuous.* Journal of Financial Economics.

[25] Kou, S. (2002). *A Jump Diffusion Model for Option Pricing.* Management Science.

[26] Heston, S. (1993). *A Closed Form Solution for Options with Stochastic Volatility.* Review of Financial Studies.

[27] Gatheral, J., Jaisson, T., and Rosenbaum, M. (2018). *Volatility is Rough.* Quantitative Finance.

[28] Mandelbrot, B. (1963). *The Variation of Certain Speculative Prices.* Journal of Business.

[29] Cont, R. (2001). *Empirical Properties of Asset Returns.* Quantitative Finance.

[30] Hamilton, J. (1989). *A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle.* Econometrica.

[31] Bouchaud, J., Bonart, J., Donier, J., and Gould, M. (2019). *Trades, Quotes and Prices.* Cambridge University Press.