

The Mathematics Behind Interest Rate Models in Decentralized Lending Protocols

Research Article

By: Amy O. Khaldoun

March 2026

Abstract

This paper presents a mathematical analysis of interest rate models used in decentralized finance lending protocols, with primary emphasis on Aave V3 and Morpho Blue. The discussion focuses on utilization based pricing, piecewise linear kinked interest rate curves, and the transmission from borrowing rates to supply rates. The paper also examines parameter choice, governance and market design trade offs, manipulation risks, and selected alternatives proposed in the DeFi literature. Worked examples illustrate how these frameworks balance liquidity incentives, capital efficiency, and solvency protection.

Keywords: DeFi, interest rate models, lending protocols, kinked curves, utilization ratio, mathematical finance

Contents

1	Introduction	4
2	Foundational Concepts	4
2.1	Utilization Ratio	4
2.2	Borrowing and Supply Rates	5
2.3	Variable and Stable Rates	5
3	Aave V3 and the Two Slope Kinked Model	5
3.1	Model Architecture	5
3.2	Borrowing Rate Formula	5
3.3	Supply Rate Formula	6
3.4	Illustrative Parameter Set	6
3.5	Worked Examples	6
3.5.1	Low Utilization: $U = 30\%$	6
3.5.2	At the Kink: $U = 90\%$	7
3.5.3	High Utilization: $U = 98\%$	7
3.6	Mathematical Properties	8
3.6.1	Continuity	8
3.6.2	Derivative Change at the Kink	9
3.6.3	Piecewise Linearity	9
4	Morpho Blue and Customizable Interest Rate Models	9
4.1	Design Philosophy	9
4.2	Borrowing Rate Formula	10
4.3	Supply Rate	10
4.4	Illustrative Morpho Style Market	10
5	Comparative Analysis of Aave and Morpho	11
5.1	Structural Comparison	11
5.2	Economic Interpretation	11
6	Parameter Sensitivity and Economic Effects	11
6.1	Effect of the Base Rate	11
6.2	Effect of the Optimal Utilization	12
6.3	Effect of Slope 1	12
6.4	Effect of Slope 2	12
6.5	Effect of the Reserve Factor	12
7	Hedging Strategies in DeFi Lending and Borrowing	12
7.1	Interest Rate Exposure	12
7.2	Delta Hedging of Collateral Exposure	13
7.3	Leveraged Looping Strategies	13
7.4	Cross Protocol Rate Arbitrage	14

7.5	Risk Management Considerations	14
8	Manipulation and Risk Considerations	15
8.1	Governance and Parameter Manipulation	15
8.2	Kink Relocation	15
8.3	Stress and Liquidity Spirals	15
8.4	Flash Loan Related Concerns	15
8.5	Mitigation Mechanisms	15
9	Alternative Interest Rate Models in DeFi	16
9.1	Linear Models	16
9.2	Polynomial Models	16
9.3	Risk Adjusted Models	16
9.4	Peer to Peer Rate Discovery	16
10	Advantages and Limitations of Kinked Models	17
10.1	Advantages	17
10.2	Limitations	17
11	A Simple Dynamic Perspective	17
12	Future Directions	18
12.1	Adaptive Utilization Targets	18
12.2	Multi Factor Models	18
12.3	Cross Protocol Feedback	18
12.4	Learning Based Calibration	18
13	Conclusion	18

1 Introduction

Decentralized finance has transformed the way interest rates are determined in lending markets. In traditional finance, benchmark rates are shaped by central banks, interbank markets, and regulated intermediaries. In DeFi, by contrast, lending protocols generally determine rates algorithmically as a function of market conditions inside each pool or market.

Among the most influential lending systems are Aave and Morpho. Aave V3 relies on governance approved utilization based interest rate strategies, while Morpho Blue introduces permissionless market creation with customizable market configurations. Both systems use utilization sensitive pricing to coordinate liquidity supply and borrowing demand.

Understanding these models matters for several groups. Protocol designers need mathematically tractable and gas efficient functions. Risk managers need to understand how rates react during stress. Suppliers need to evaluate expected yield. Borrowers need to estimate funding costs under changing utilization.

This paper develops the core mathematical framework behind these systems and studies the economic logic embedded in their parameters. The discussion proceeds from foundational definitions to model construction, comparative analysis, practical examples, and limitations.

The main contributions are as follows:

- formal derivation of utilization based borrowing and supply rates
- analysis of the two slope kinked curve structure
- comparison of Aave V3 and Morpho Blue market design
- discussion of parameter sensitivity and manipulation risks
- review of alternative approaches in DeFi lending

2 Foundational Concepts

2.1 Utilization Ratio

The central state variable in most DeFi lending protocols is the utilization ratio. It measures how much of supplied capital is currently borrowed. A common definition is

$$U = \frac{B}{S + B}, \tag{1}$$

where

- B denotes total borrowed amount,
- S denotes currently idle supplied liquidity,
- $U \in [0, 1]$ is the fraction of total supplied capital that is actively borrowed.

If total supplied capital is denoted by $L = S + B$, then the same quantity can be written as

$$U = \frac{B}{L}. \tag{2}$$

Economically, low utilization indicates abundant idle liquidity and weak demand for borrowing. High utilization indicates tight liquidity and strong demand for borrowed capital. Because utilization summarizes scarcity inside the market, it is the natural input for an on chain pricing rule.

2.2 Borrowing and Supply Rates

A lending protocol usually tracks two related rates:

- the **borrowing rate**, paid by borrowers,
- the **supply rate**, earned by liquidity providers.

The borrowing rate is determined by the protocol’s interest rate model. The supply rate is then derived from borrowing activity, utilization, and any reserve factor or protocol fee. The exact link depends on market design, but the broad logic is simple. Suppliers can only earn when capital is borrowed, so supply yield scales with both utilization and the borrowing rate.

2.3 Variable and Stable Rates

Some protocols distinguish between variable and stable borrowing modes. Variable rates move with utilization and market conditions. Stable rates attempt to smooth funding costs over time, though they are often not perfectly fixed in the strict traditional finance sense.

Type	Key Feature	Typical Use
Variable	Moves with utilization	Short horizon borrowing
Stable	More predictable over time	Cost planning

Table 1: Stylized distinction between variable and stable borrowing modes.

This paper focuses on variable interest rate models, since they are the core mechanism in Aave V3 and Morpho Blue.

3 Aave V3 and the Two Slope Kinked Model

3.1 Model Architecture

Aave V3 uses a piecewise linear borrowing rate curve with a kink at an optimal utilization threshold. This structure is designed to keep utilization in a desirable region. When utilization is below the target, rates rise gradually. Once utilization exceeds the target, rates rise much faster in order to discourage additional borrowing and attract more supply.

3.2 Borrowing Rate Formula

Let

- r_0 be the base borrowing rate,

- U^* be the optimal utilization ratio,
- m_1 be the slope below the kink,
- m_2 be the slope above the kink.

Then the variable borrowing rate is

$$r_b(U) = \begin{cases} r_0 + \frac{U}{U^*}m_1, & \text{if } U \leq U^*, \\ r_0 + m_1 + \frac{U - U^*}{1 - U^*}m_2, & \text{if } U > U^*. \end{cases} \quad (3)$$

This function is continuous at $U = U^*$ but its derivative changes discretely there. That derivative change is the mathematical meaning of the kink.

3.3 Supply Rate Formula

If the reserve factor is denoted by R , then only a fraction $(1 - R)$ of interest paid by borrowers is passed to suppliers. The supply rate is

$$r_s(U) = r_b(U) U (1 - R). \quad (4)$$

This expression captures two facts. First, supplier yield is higher when borrowers pay more. Second, supplier yield is also higher when a larger share of the pool is actively borrowed.

3.4 Illustrative Parameter Set

A stylized parameter set for a stablecoin market can be written as follows.

Param.	Symbol	Value
Base rate	r_0	0%
Opt. util.	U^*	90%
Slope 1	m_1	9.5%
Slope 2	m_2	75%
Reserve factor	R	10%

Table 2: Illustrative Aave style parameters for a stablecoin market.

3.5 Worked Examples

3.5.1 Low Utilization: $U = 30\%$

Since $U < U^*$,

$$r_b(0.30) = 0 + \frac{0.30}{0.90} \times 0.095 = 0.0317 \approx 3.17\%. \quad (5)$$

The supply rate is

$$r_s(0.30) = 0.0317 \times 0.30 \times 0.90 = 0.00856 \approx 0.86\%. \quad (6)$$

3.5.2 At the Kink: $U = 90\%$

$$r_b(0.90) = 0 + \frac{0.90}{0.90} \times 0.095 = 0.095 = 9.5\%. \quad (7)$$

$$r_s(0.90) = 0.095 \times 0.90 \times 0.90 = 0.07695 \approx 7.70\%. \quad (8)$$

3.5.3 High Utilization: $U = 98\%$

Since $U > U^*$,

$$r_b(0.98) = 0 + 0.095 + \frac{0.98 - 0.90}{1 - 0.90} \times 0.75. \quad (9)$$

Therefore,

$$r_b(0.98) = 0.095 + 0.8 \times 0.75 = 0.095 + 0.60 = 0.695 = 69.5\%. \quad (10)$$

The supply rate is

$$r_s(0.98) = 0.695 \times 0.98 \times 0.90 = 0.61299 \approx 61.3\%. \quad (11)$$

These examples show the policy intention embedded in the curve. Below the target, borrowing remains relatively cheap. Above the target, rates rise sharply in order to protect residual liquidity.

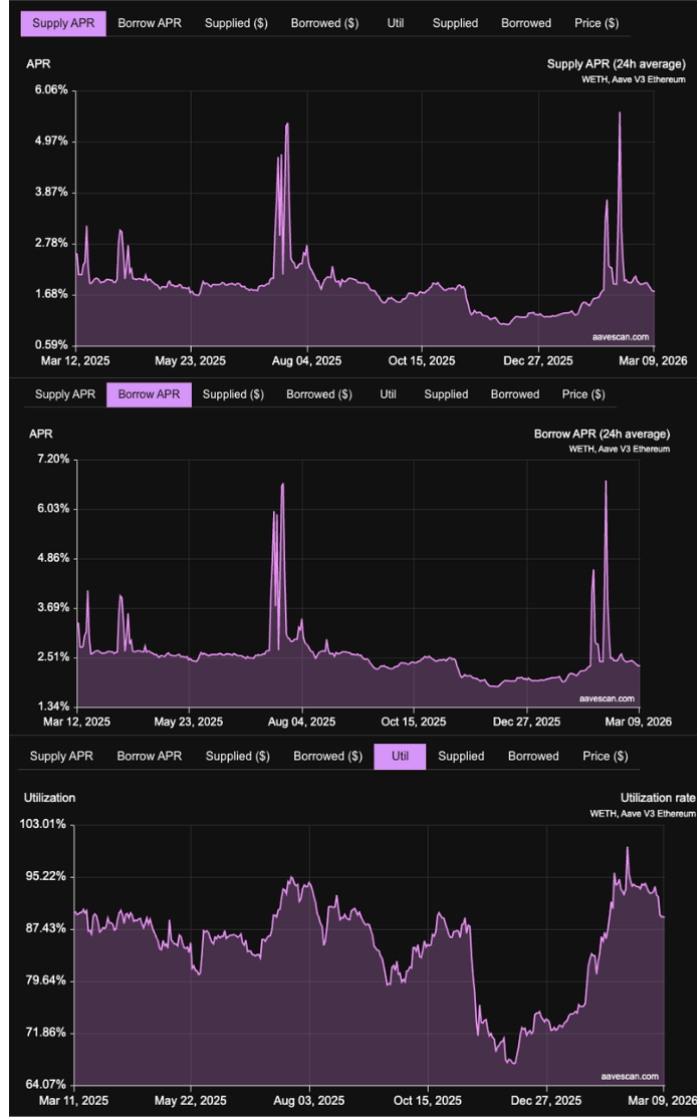


Figure 1: Historical supply APR, borrow APR, and utilization for the WETH market on Aave V3 (Ethereum) over a one year period. The figure illustrates the interaction between utilization and interest rates in practice. Periods of elevated utilization correspond to spikes in borrowing costs and higher supply yields, reflecting the utilization based interest rate mechanism implemented in Aave’s kinked rate model. Source: AaveScan.

3.6 Mathematical Properties

3.6.1 Continuity

The function is continuous at U^* because

$$\lim_{U \rightarrow U^{*-}} r_b(U) = r_0 + m_1 = \lim_{U \rightarrow U^{*+}} r_b(U). \quad (12)$$

3.6.2 Derivative Change at the Kink

For $U < U^*$,

$$\frac{\partial r_b}{\partial U} = \frac{m_1}{U^*}. \quad (13)$$

For $U > U^*$,

$$\frac{\partial r_b}{\partial U} = \frac{m_2}{1 - U^*}. \quad (14)$$

Using the illustrative values above,

$$\left. \frac{\partial r_b}{\partial U} \right|_{U < 0.90} = \frac{0.095}{0.90} \approx 0.1056, \quad (15)$$

and

$$\left. \frac{\partial r_b}{\partial U} \right|_{U > 0.90} = \frac{0.75}{0.10} = 7.5. \quad (16)$$

The slope above the kink is therefore much steeper than the slope below it.

3.6.3 Piecewise Linearity

It is important to state the geometry correctly. The function is piecewise linear, not strictly convex over each interval. Its second derivative is zero away from the kink and undefined at the kink. The economically relevant property is not smooth convexity, but rather the discrete increase in marginal borrowing cost once utilization exceeds the threshold.

4 Morpho Blue and Customizable Interest Rate Models

4.1 Design Philosophy

Morpho Blue departs from the single pool architecture by allowing permissionless market creation. Each market is associated with a collateral asset, a loan asset, a loan to value parameter, an oracle, and an interest rate model. This makes market design modular and more flexible than a one size fits all pool.

A consequence of this modular architecture is that empirical analysis of interest rates differs from pooled lending protocols such as Aave. Aave maintains a single liquidity pool per asset, which produces a unique utilization level and a single borrowing rate for that market. This makes it straightforward to visualize historical borrowing rates, supply rates, and utilization dynamics at the pool level.

In contrast, Morpho Blue consists of many isolated markets defined by specific collateral–loan asset pairs. Each market has its own utilization ratio, liquidity distribution, and interest rate model. As a result, there is no single global borrowing rate for a given asset, and empirical analysis typically focuses on individual markets rather than the protocol as a whole.

4.2 Borrowing Rate Formula

A simple Morpho style kinked model can be expressed with the same piecewise linear form:

$$r_b^{\text{Morpho}}(U) = \begin{cases} r_0 + \frac{U}{U^*} m_1, & \text{if } U \leq U^*, \\ r_0 + m_1 + \frac{U - U^*}{1 - U^*} m_2, & \text{if } U > U^*. \end{cases} \quad (17)$$

The functional form is similar, but the surrounding market design differs. In particular:

- markets are created permissionlessly,
- parameters can differ across markets,
- the architecture is modular,
- suppliers can face a wider range of market specific conditions.

4.3 Supply Rate

In a simplified no reserve factor representation, the supply rate is

$$r_s^{\text{Morpho}}(U) = r_b(U) U. \quad (18)$$

This means the full interest flow from borrowers is passed to suppliers within the market, abstracting from any other market level frictions or external costs.

4.4 Illustrative Morpho Style Market

Consider a stylized ETH USDC market with the following parameters.

Param.	Symbol	Value
Base rate	r_0	0.5%
Opt. util.	U^*	85%
Slope 1	m_1	4%
Slope 2	m_2	80%
Reserve factor	R	0%

Table 3: Illustrative Morpho style market parameters.

If utilization is $U = 75\%$, then

$$r_b(0.75) = 0.005 + \frac{0.75}{0.85} \times 0.04 = 0.005 + 0.0353 = 0.0403 \approx 4.03\%. \quad (19)$$

The supply rate is

$$r_s(0.75) = 0.0403 \times 0.75 = 0.0302 \approx 3.02\%. \quad (20)$$

If an otherwise identical market imposed a 10% reserve factor, then the supply rate would become

$$r_s^{\text{fee}}(0.75) = 0.0403 \times 0.75 \times 0.90 = 0.0272 \approx 2.72\%. \quad (21)$$

The difference highlights how fee extraction affects supplier incentives.

5 Comparative Analysis of Aave and Morpho

5.1 Structural Comparison

Aave and Morpho can use similar rate curves, but they differ in market organization, governance structure, and economic flexibility.

Aspect	Aave V3	Morpho Blue
Market design	Pool based	Modular market based
Parameters	Governance chosen	Market specific setup
Reserve factor	Usually positive	Can be zero in stylized form
IRM flexibility	More standardized	More customizable
Permissionless markets	No	Yes

Table 4: High level comparison of Aave V3 and Morpho Blue.

5.2 Economic Interpretation

Aave emphasizes coordinated risk management and consistency across listed assets. This can improve standardization and reduce user confusion. Morpho emphasizes granularity and flexibility. This can improve fit for specific asset pairs and collateral profiles, but it also increases heterogeneity across markets.

From a supplier’s perspective, a lower fee take generally supports higher yield, holding utilization and borrower demand fixed. From a borrower’s perspective, the relevant question is how quickly the rate rises as liquidity becomes scarce. That depends on the choice of U^* , m_1 , and m_2 .

6 Parameter Sensitivity and Economic Effects

6.1 Effect of the Base Rate

The base rate r_0 shifts the entire curve upward:

$$\frac{\partial r_b}{\partial r_0} = 1. \quad (22)$$

A higher base rate can discourage borrowing even when utilization is low. This may be desirable for riskier markets or for assets that should not be borrowed too cheaply.

6.2 Effect of the Optimal Utilization

Changing U^* changes both the location of the kink and the scaling of the linear segments. A higher U^* delays the point at which punitive pricing begins. A lower U^* makes the system defensive earlier.

6.3 Effect of Slope 1

The parameter m_1 controls how aggressively rates rise below the kink. A larger m_1 makes borrowing more expensive even in normal conditions.

6.4 Effect of Slope 2

The parameter m_2 controls the severity of the response once utilization exceeds the target. A larger m_2 can make the market highly resistant to full utilization by making marginal borrowing extremely expensive during stress.

6.5 Effect of the Reserve Factor

The reserve factor does not directly alter the borrower's quoted rate under the simple formulation above, but it reduces pass through to suppliers:

$$\frac{\partial r_s}{\partial R} = -r_b(U)U < 0. \quad (23)$$

Thus a higher reserve factor lowers supplier yield for any given utilization and borrowing rate.

7 Hedging Strategies in DeFi Lending and Borrowing

Participants in decentralized lending markets are exposed to several forms of risk. Liquidity providers face fluctuations in supply yields as utilization changes, while borrowers face uncertainty regarding future borrowing costs. In addition, positions that involve volatile collateral introduce market price risk, since large movements in asset prices can trigger liquidations. As a result, many users employ hedging strategies in order to stabilize returns or reduce exposure to adverse price movements.

This section outlines the main sources of risk in DeFi lending markets and discusses common techniques used to hedge these exposures.

7.1 Interest Rate Exposure

Borrowing rates in decentralized lending protocols depend on the utilization ratio of the market. Because utilization evolves dynamically as liquidity conditions change, borrowing costs may fluctuate significantly over time.

Consider a borrower with outstanding debt D at borrowing rate $r_b(U)$. The instantaneous interest cost is

$$C = D \cdot r_b(U). \tag{24}$$

If utilization increases sharply, particularly when the market approaches or exceeds the optimal utilization threshold U^* , borrowing costs can rise substantially. Borrowers therefore face interest rate risk analogous to floating rate borrowing in traditional financial markets.

One simple approach to reducing interest rate exposure is diversification across multiple lending protocols. Suppose a borrower allocates debt across n protocols with weights w_i , where $\sum_{i=1}^n w_i = 1$. The effective borrowing rate becomes

$$r_{\text{eff}} = \sum_{i=1}^n w_i r_{b,i}. \tag{25}$$

Because utilization dynamics differ across protocols and markets, diversification can reduce the sensitivity of borrowing costs to shocks in any single lending market.

7.2 Delta Hedging of Collateral Exposure

Many DeFi lending positions involve depositing a volatile asset as collateral while borrowing a stable asset. For example, a user may deposit ETH as collateral and borrow USDC. In such cases, the user becomes exposed to fluctuations in the price of the collateral asset.

Let P denote the price of the collateral asset and Q the amount deposited. The value of the collateral position is

$$V = PQ. \tag{26}$$

If the collateral price declines significantly, the value of the position may approach the liquidation threshold imposed by the protocol. To hedge this exposure, users often open a short position in the same asset using derivatives such as perpetual futures or options.

If the user opens a short derivative position of size Q , the payoff from the hedge is approximately

$$\Delta V_{\text{hedge}} = -Q\Delta P. \tag{27}$$

This offsets the change in collateral value

$$\Delta V_{\text{collateral}} = Q\Delta P. \tag{28}$$

The combined position therefore becomes approximately delta neutral with respect to small price changes in the underlying asset.

7.3 Leveraged Looping Strategies

A widely used strategy in decentralized lending markets is recursive borrowing and supplying, commonly referred to as *looping*. In this strategy, a user repeatedly supplies an asset as collateral, borrows against that collateral, and then supplies the borrowed asset again. The process increases the effective supplied capital and therefore amplifies exposure to lending yields.

Suppose a user initially supplies capital S and is able to borrow a fraction λS , where λ represents the loan-to-value ratio of the position. If the borrowed funds are supplied again and the process is repeated multiple times, the total supplied capital becomes

$$S_{\text{total}} = S \sum_{i=0}^k \lambda^i. \quad (29)$$

As the number of loops increases, this geometric series converges to

$$S_{\text{total}} = \frac{S}{1 - \lambda}. \quad (30)$$

Similarly, the total borrowed capital approaches

$$B_{\text{total}} = \frac{\lambda S}{1 - \lambda}. \quad (31)$$

The net interest return from the position can be expressed as

$$\Pi = S_{\text{total}} r_s(U) - B_{\text{total}} r_b(U), \quad (32)$$

where $r_s(U)$ and $r_b(U)$ denote the supply and borrowing rates determined by the utilization based interest rate model. Looping strategies are profitable when the effective supply yield exceeds the borrowing cost, or when additional incentives such as liquidity mining rewards are present.

Although looping can significantly increase yield, it also amplifies risk. The effective leverage of the position increases with the number of loops, making the position more sensitive to interest rate fluctuations and to price movements of the collateral asset.

7.4 Cross Protocol Rate Arbitrage

Another strategy used by sophisticated participants involves exploiting differences in interest rates across protocols. If borrowing costs on one platform are lower than supply yields on another, users may borrow on the cheaper protocol and supply funds to the higher yielding one.

Let $r_b^{(A)}$ denote the borrowing rate on protocol A and $r_s^{(B)}$ the supply rate on protocol B . An arbitrage opportunity exists when

$$r_s^{(B)} > r_b^{(A)}. \quad (33)$$

In practice, such opportunities may arise temporarily due to differences in utilization, liquidity conditions, or parameter settings across protocols. However, these opportunities tend to disappear quickly as capital flows between markets.

7.5 Risk Management Considerations

Although hedging strategies can reduce exposure to certain risks, they also introduce additional complexities. Derivative hedges involve funding costs and margin requirements, while cross protocol strategies depend on liquidity conditions and execution speed. Leveraged looping strategies amplify both interest rate risk and liquidation risk.

Consequently, effective risk management in DeFi lending requires continuous monitoring of utilization ratios, collateral prices, and protocol specific parameters such as loan to value limits and liquidation thresholds. Participants must balance the potential yield benefits of leverage and arbitrage strategies against the increased risk of adverse market movements.

8 Manipulation and Risk Considerations

8.1 Governance and Parameter Manipulation

If protocol governance controls rate parameters, then rate policy becomes a governance object. In principle, malicious or careless governance decisions could distort incentives.

For example, a reduction in slope below the kink lowers borrowing cost for a given utilization level:

$$\Delta r_b = \frac{U}{U^*} \Delta m_1, \quad U \leq U^*. \quad (34)$$

If $\Delta m_1 < 0$, the rate falls. A similar statement applies above the kink for changes in m_2 .

8.2 Kink Relocation

Changing U^* can postpone or accelerate stress pricing. If U^* is increased, then the market tolerates higher utilization before the steep segment starts. This can be attractive in calm conditions but dangerous during rapid withdrawals.

8.3 Stress and Liquidity Spirals

High utilization can create a feedback loop. As utilization rises, borrowing rates rise. If rates rise sharply, some borrowers repay. However, if the market is already strained and liquidity providers are slow to react, the pool can remain close to full utilization. In such states, rollover risk and liquidation pressure can increase together.

8.4 Flash Loan Related Concerns

A large temporary capital movement can sharply alter utilization within a block or over a short horizon. If other mechanisms depend on current utilization, this can create exploitable timing effects. Whether such an effect is profitable depends on implementation details, oracle timing, liquidation rules, and interest accrual mechanics.

8.5 Mitigation Mechanisms

Common mitigation ideas include:

- governance timelocks,
- parameter caps and floors,
- conservative slope calibration,
- market specific risk reviews,

- transparent monitoring dashboards.

A generic cap framework can be written as

$$m_1 \in [m_1^{\min}, m_1^{\max}], \quad m_2 \in [m_2^{\min}, m_2^{\max}], \quad U^* \in [U_{\min}^*, U_{\max}^*]. \quad (35)$$

9 Alternative Interest Rate Models in DeFi

9.1 Linear Models

The simplest alternative is a single slope linear rule:

$$r_b(U) = a + bU. \quad (36)$$

This is easy to implement but does not sharply penalize near exhaustion of liquidity.

9.2 Polynomial Models

A more flexible family is

$$r_b(U) = a + bU + cU^2. \quad (37)$$

This can create a smoother convex increase in borrowing costs, though it may be less intuitive for users and may require more careful calibration.

9.3 Risk Adjusted Models

Some proposed designs add explicit risk terms:

$$r_b = f(U, \sigma, LTV, q), \quad (38)$$

where σ represents volatility, LTV is the loan to value ratio, and q may summarize collateral quality or liquidation risk. Such models are richer but more complex operationally.

9.4 Peer to Peer Rate Discovery

Other systems seek to move away from a purely formulaic pool rate and instead allow lenders and borrowers to meet through a more market driven process. This can improve price discovery in some cases, but it may reduce simplicity and immediate liquidity.

Property	Kinked	Linear	Poly.	P2P
Gas cost	Low	Very low	Medium	Higher
Transparency	High	High	Medium	Lower
Stress response	Strong	Weak	Medium	Market driven
Calibration ease	Medium	High	Lower	Lower

Table 5: Stylized comparison of alternative interest rate model families.

10 Advantages and Limitations of Kinked Models

10.1 Advantages

Kinked models offer several practical benefits.

First, they are easy to implement and explain. The formula is simple, piecewise linear, and computationally efficient.

Second, they embed a clear policy objective. The model tolerates normal borrowing activity below the target and responds strongly when liquidity becomes scarce.

Third, they are parameterized in a way that is intuitive for governance and risk teams. Each parameter has a recognizable interpretation.

10.2 Limitations

The same simplicity also creates limitations.

First, the change in slope at the kink is abrupt. That can generate sudden jumps in marginal funding cost near U^* .

Second, utilization is an incomplete state variable. It does not directly encode asset volatility, liquidation depth, oracle reliability, or broader market stress.

Third, a fixed optimal utilization may not suit every asset class or market regime. A stable-coin market and a volatile long tail asset market may require very different behavior.

11 A Simple Dynamic Perspective

To formalize the balancing logic, suppose utilization evolves according to

$$\frac{dU}{dt} = \alpha(D(r_b(U)) - S(r_s(U))), \quad (39)$$

where

- D is borrowing demand, decreasing in the borrowing rate,
- S is liquidity supply, increasing in the supply rate,
- $\alpha > 0$ is an adjustment speed constant.

An equilibrium utilization \bar{U} satisfies

$$D(r_b(\bar{U})) = S(r_s(\bar{U})). \quad (40)$$

The key insight is that the interest rate function provides negative feedback. If utilization rises, borrowing becomes more expensive and supply becomes more attractive. If utilization falls, the opposite occurs. The kink strengthens this negative feedback once the market enters a stressed region.

This argument is qualitative rather than a full theorem, since global stability depends on the exact shapes of D and S . Still, it captures the central economic purpose of utilization based interest rate engineering.

12 Future Directions

Several extensions to current DeFi rate models are plausible.

12.1 Adaptive Utilization Targets

One possible extension is to let the optimal utilization vary over time:

$$U^*(t) = g(\sigma_t, TVL_t, \text{depth}_t). \quad (41)$$

This would make the location of the kink state dependent rather than fixed.

12.2 Multi Factor Models

Another direction is to allow rates to depend on several risk signals at once:

$$r_b = f(U, \sigma, LTV, \lambda), \quad (42)$$

where λ may represent liquidity stress or market fragmentation.

12.3 Cross Protocol Feedback

As capital moves across DeFi venues, rates on one protocol can influence supply and borrowing conditions on another. This can lead to arbitrage, migration of liquidity, and more tightly coupled ecosystem level dynamics.

12.4 Learning Based Calibration

Machine learning methods may eventually be used to assist parameter calibration, though any such framework would need strong constraints, transparency, and robust fail safe design before it could be trusted on chain.

13 Conclusion

Interest rate models in DeFi lending protocols translate liquidity conditions into borrowing costs and supplier yield. Aave V3 and Morpho Blue illustrate how a relatively simple mathematical structure can perform a complex economic coordination task.

The core object is the utilization sensitive borrowing rate. In the kinked model, rates rise gradually when liquidity is abundant and sharply when liquidity becomes scarce. This generates negative feedback that helps prevent persistent pool exhaustion.

Aave emphasizes standardized governance controlled deployment of these mechanisms. Morpho Blue emphasizes modularity and market level flexibility. The underlying mathematics can look very similar, but the surrounding market architecture changes the economic consequences.

Kinked models remain attractive because they are transparent, efficient, and easy to parameterize. Their main weakness is that they reduce market state to a small set of variables,

especially utilization. As DeFi matures, richer models may incorporate additional risk signals, but the trade off between sophistication and transparency will remain central.

Acknowledgments

This article synthesizes public protocol documentation, research commentary, and academic discussion on decentralized lending and utilization based pricing.

References

- [1] Aave Protocol. *Aave V3 Smart Contracts: Interest Rate Strategy*. Available at: <https://aave.com/docs/aave-v3/smart-contracts/interest-rate-strategy>
- [2] Hacken. *Aave Borrow Rate Tuning: A Practical Guide*. Available at: <https://hacken.io/insights/aave-borrow-rate-tuning/>
- [3] Kupia Security. *Decoding Aave's Interest Rate Engine: A Developer's and Security Researcher's Guide to Risk*. Available at: <https://medium.com/@kupiasec/decoding-aaves-interest-rate-engine-a-developer-s-and-security-researcher-s-guide-to-risk>
- [4] Krayon Digital. *Aave Interest Rate Model Explained*. Available at: <https://www.krayondigital.com/blog/aave-interest-rate-model-explained>
- [5] LlamaRisk. *AAVE Interest Rate Model and the TradFi Symbiosis*. Available at: <https://www.llamarisk.com/research/aave-irm>
- [6] Morpho Labs. *Interest Rate Models*. Available at: <https://docs.morpho.org/get-started/resources/contracts/irm/>
- [7] Morpho Labs. *Interest Rate Model Concepts*. Available at: <https://docs.morpho.org/learn/concepts/irm/>
- [8] Morpho Labs. *Get Data: Interest Rate Model Calculations*. Available at: <https://docs.morpho.org/build/borrow/tutorials/get-data>
- [9] MixBytes. *Modern DeFi Lending Protocols: How It's Made, Morpho Blue*. Available at: <https://mixbytes.io/blog/modern-defi-lending-protocols-how-its-made-morpho-blue>
- [10] Bertucci et al. *Agents' Behavior and Interest Rate Model Optimization in DeFi*. Available at: <https://onlinelibrary.wiley.com/doi/full/10.1111/mafi.70002>
- [11] Anonymous. *Optimal Risk Aware Interest Rates for Decentralized Lending Protocols*. Available at: <https://arxiv.org/html/2502.19862v1>
- [12] Banque de France. *Interest Rates in Decentralised Finance*. Available at: https://www.banque-france.fr/system/files/2024-04/Billet_352_EN.pdf
- [13] Morpho Labs. *Morpho V1 Yellow Paper*. Available at: <https://hal.science/hal-04087388v1/file/yellowpaper.morpho.xyz.pdf>
- [14] Binance Research. *A Quantitative Lens on Morpho vs. Pooled Lending*. Available at: <https://www.binance.com/en-IN/square/post/32534291271282>